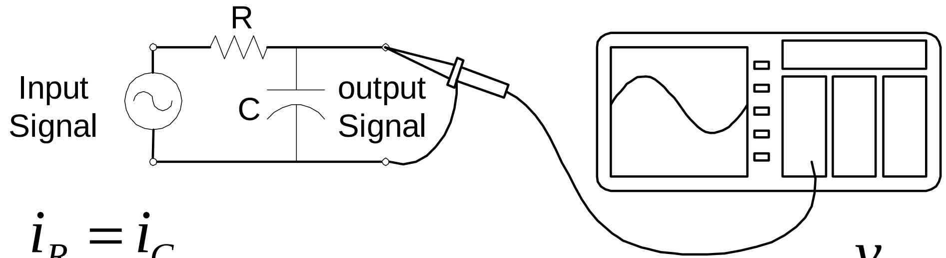
Signal Filtering

Passive filters

First Order Low Pass Filter

- Consider the following circuit
  - A signal with a desired signal and an interference signal may be supplied. The interference is at a high frequency in this case and the desired at a low frequency:



$$i_R = i_C$$

$$i_R = \frac{v_R}{R}$$

$$i_C = C\frac{dv_{out}}{dt}$$

$$v_R = v_{in} - v_C = v_{in} - v_{out}$$

$$\frac{v_R}{R} = C\frac{dv_{out}}{dt}$$

$$\frac{v_{in} - v_{out}}{R} = C\frac{dv_{out}}{dt}$$

$$v_{in} = RC\frac{dv_{out}}{dt} + v_{out}$$

First Order Low Pass Filter

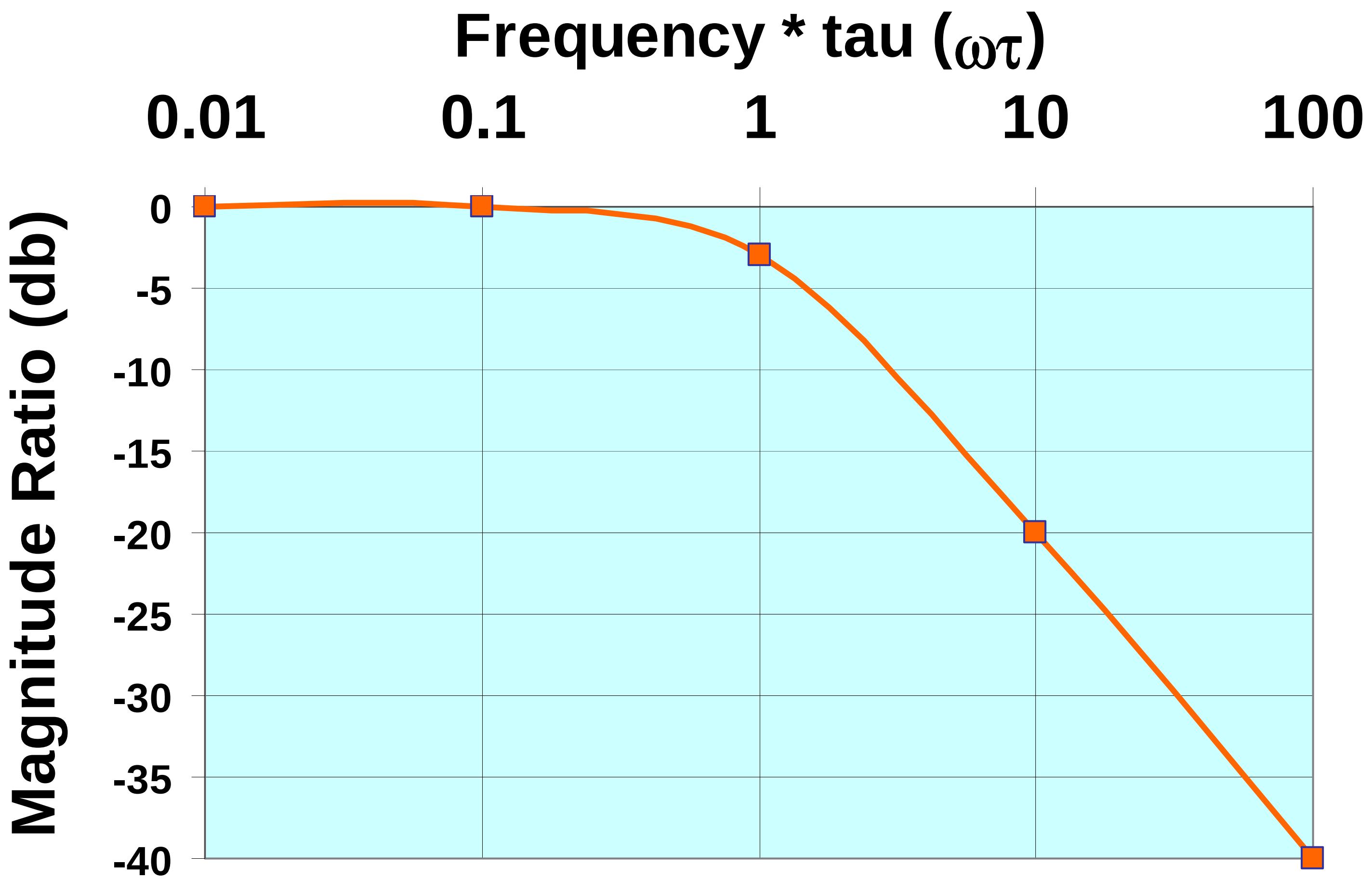
- Assuming deviation variables:

$$v_{in}(s) = RCsv_{out}(s) + v_{out}(s)$$

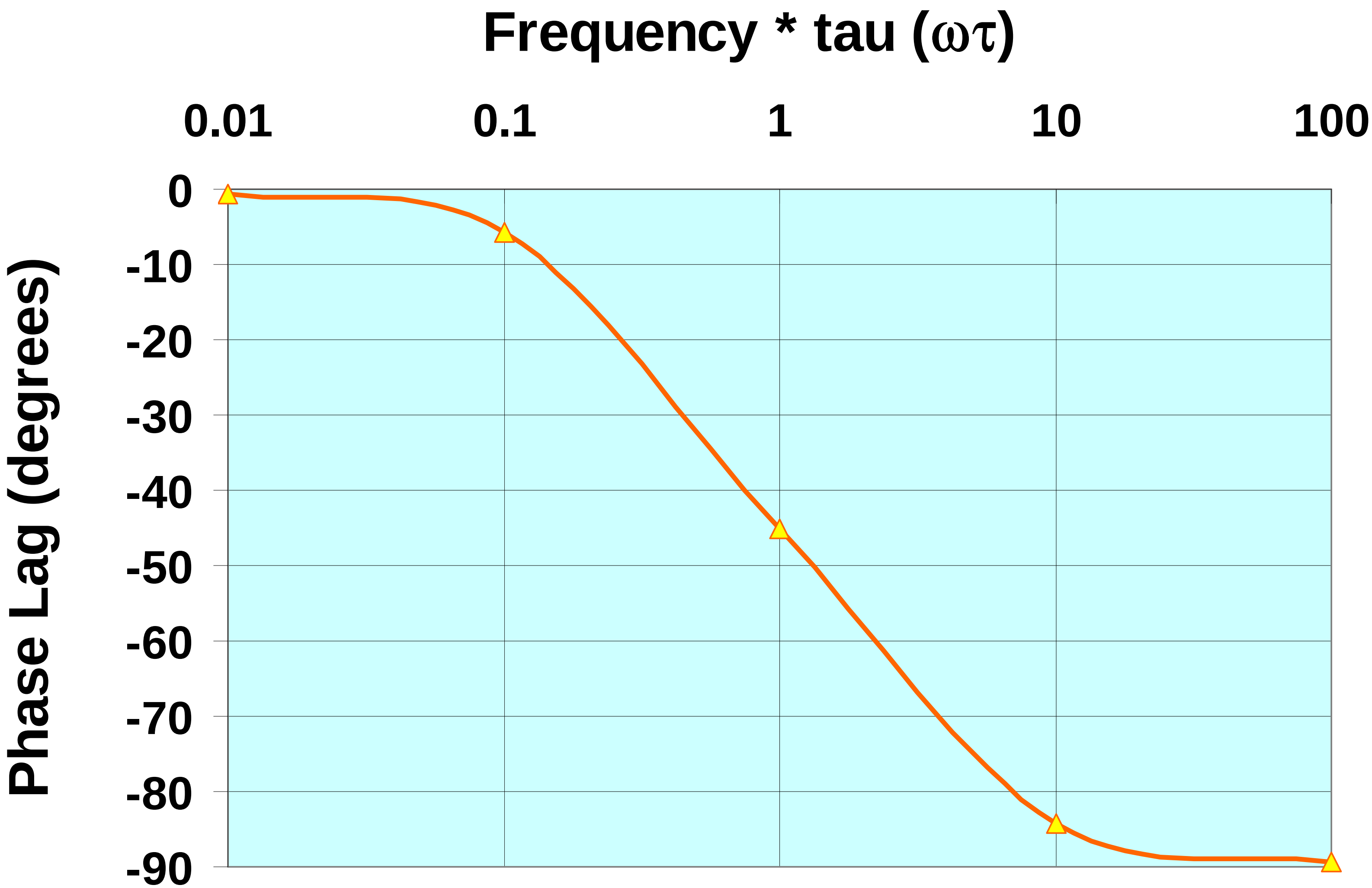
$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{1}{RCs+1} = \frac{1}{\tau s+1}$$

- First order system
  - Ratio of output to input will follow ideal response for a first order system
  - Easily calculated by substituting $i\omega$ for s and computing $|G(i\omega)|$ and $\angle G(i\omega)$

Plotting Frequency response



Plotting Frequency response



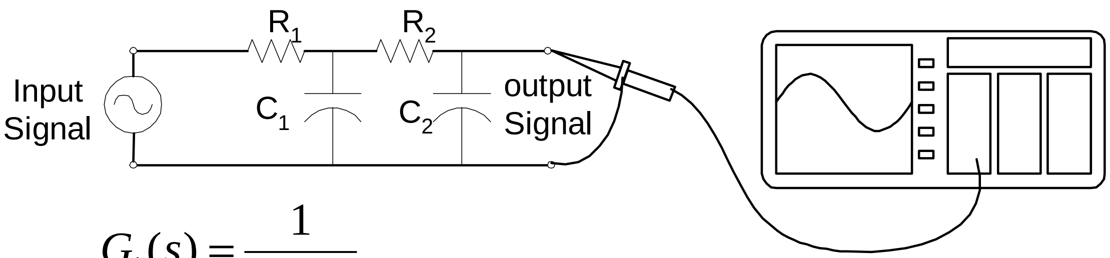
Adding terms of Frequency Response

$$20\log\left(\left|G_1(i\omega)\right|\left|G_2(i\omega)\right|\right) = 20\log\left(\left|G_1(i\omega)\right|\right) + 20\log\left(\left|G_2(i\omega)\right|\right)$$

$$\angle G_1(i\omega)G_2(i\omega) = \angle G_1(i\omega) + \angle G_2(i\omega)$$

We can simply add terms on the Bode Magnitude plot and on the Bode Phase plot to get total response

Second Order Low Pass Filter



$$G_1(s) = \frac{1}{\tau_1 s + 1}$$

$$G_2(s) = \frac{1}{\tau_2 s + 1}$$

$$G(s) = \frac{1}{\tau_1 s + 1}\frac{1}{\tau_2 s + 1}$$

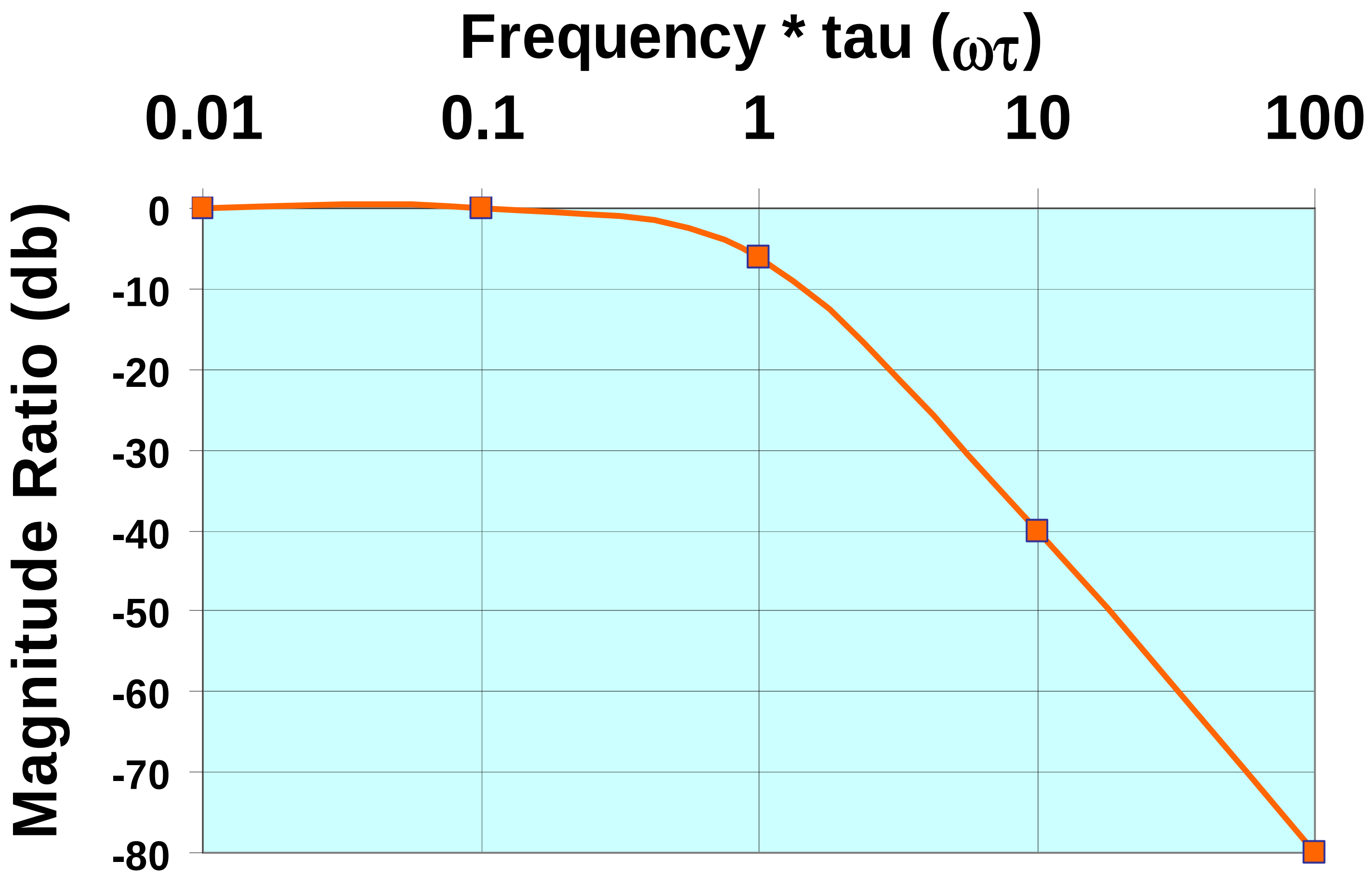
$$G(i\varpi) = \frac{1}{\tau_1 i\omega + 1}\frac{1}{\tau_2 i\omega + 1}$$

$$20\log\left|G(i\omega)\right| = 20\log\left|\frac{1}{\tau_1 i\omega + 1}\right| + 20\log\left|\frac{1}{\tau_2 i\omega + 1}\right|$$

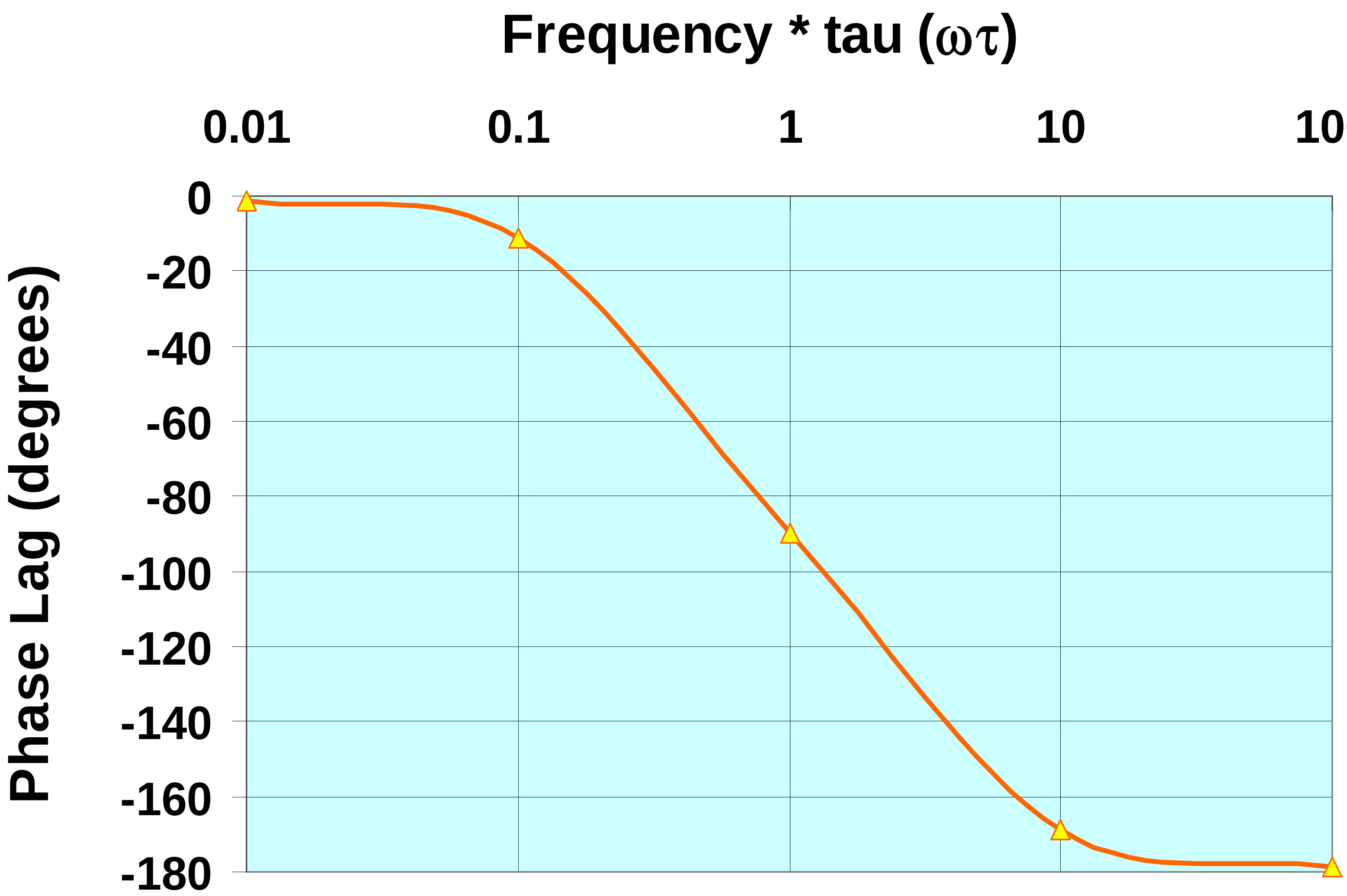
$$\angle G(i\omega) = \angle\frac{1}{\tau_1 i\omega + 1} + \angle\frac{1}{\tau_2 i\omega + 1}$$

This works as long as the assumption of no current flowing between R_1 and R_2 holds. Make $R_2 >> R_1$

Plotting Frequency response

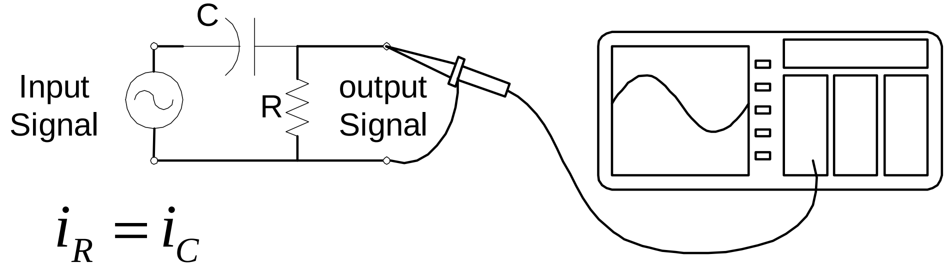


Plotting Frequency response



First Order High Pass Filter

- Consider the following circuit



$$i_R = i_C$$

$$i_R = \frac{v_{out}}{R}$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C = v_{in} - v_R = v_{in} - v_{out}$$

$$\frac{v_{out}}{R} = C\frac{dv_C}{dt}$$

$$\frac{v_{out}}{R} = C\frac{d(v_{in} - v_{out})}{dt}$$

$$RC\frac{dv_{in}}{dt} = RC\frac{dv_{out}}{dt} + v_{out}$$

$$RCsv_{in}(s) = RCsv_{out}(s) + v_{out}(s)$$

$$\frac{v_{out}(s)}{v_{in}(s)} = \frac{RCs}{RCs+1} = \frac{\tau s}{\tau s+1}$$

First Order High Pass Filter

$$G(i\omega) = \frac{\tau i\omega}{\tau i\omega + 1} = \tau i\omega\left[\frac{1}{\tau i\omega + 1}\right]$$

$$20\log|G(i\omega)| = 20\log|\tau i\omega| + 20\log\left|\frac{1}{\tau i\omega + 1}\right|$$

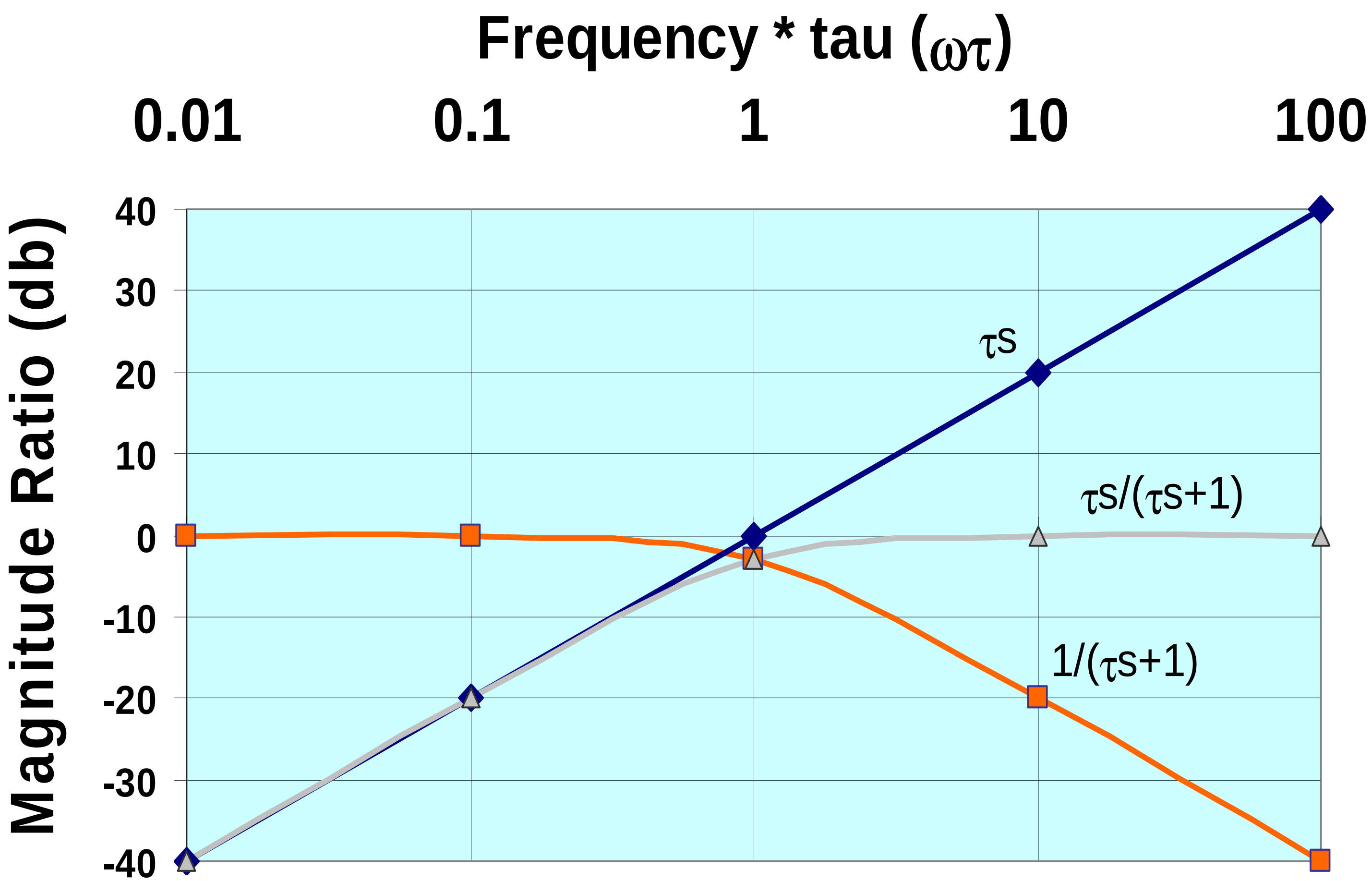
$$|\tau i\omega| = \sqrt{0 + \tau^2\omega^2} = \tau\omega$$

Freq.	$\lvert G(i\omega)\rvert$	$20\log\lvert G(i\omega)\rvert$
$.1/\tau$	0.1	-20
$1/\tau$	1	0
$10/\tau$	10	20

$$\angle G(i\omega) = \angle i\omega + \angle \frac{1}{\tau i\omega + 1}$$

$$\angle i\omega = \tan^{-1}\left(\frac{\omega}{0}\right) = +90^0$$

Plotting Frequency response



Plotting Frequency response