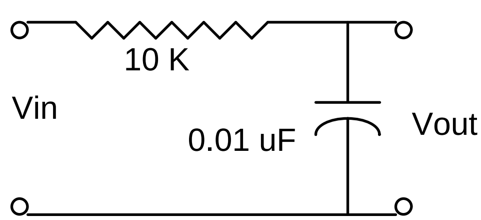
LABORATORY III
BAE 5413
SPRING 2007

TITLE: Passive Filters

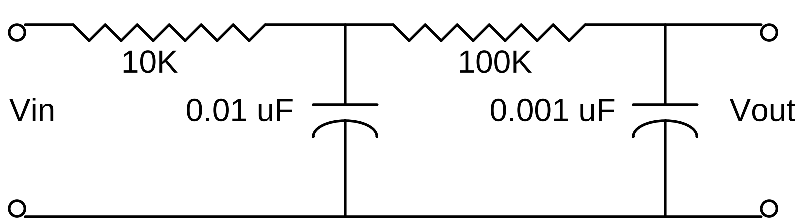
OBJECTIVE:
To experimentally determine the response of several passive filter designs and compare their response with theoretically calculated response

PROCEDURE:
Fabricate the following filter designs and experimentally determine their frequency response. For the first order low pass filter (filter 1), determine both magnitude and phase response. For the other filters, determine only the magnitude response. Plot the filter responses in the form of Bode Diagrams for theoretical and experimental data (theoretical should be plotted as a continuous only curve and experimental as data symbols only). Use measured component values to calculate the corner frequencys for the experimental response. The development attached to this handout may be useful. Determine the deviation in corner frequency of the experimental response from theoretical. Estimate the error in your measurements and determine whether the measured corner frequency falls within the range of error expected. Briefly discuss accuracy of corner frequency determination and present results for each filter in your report. Identify the type of filter for each case.

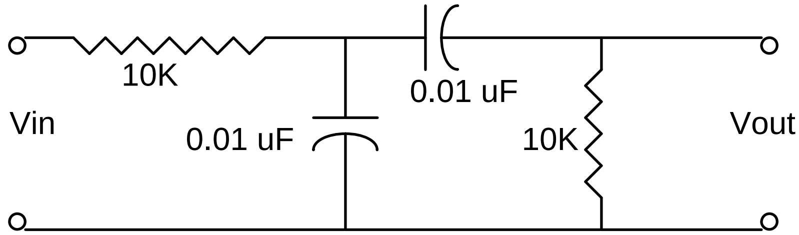
FILTER DESIGNS:



Filter 1



Filter 2



Filter 3

For a first order passive low pass filter:
Voltage equation

$$V_{in} = V_R + V_C$$

Current equation

$$I_R = I_C = I$$

Properties of the circuit elements

$$V_R = RI_R \text{, and } I_C = CdV_C /dt$$

Replacing I_R and I_C in the current equation with the properties equations:

$$V_R / R = CdV_C /dt$$

Then replacing V_R in the voltage equation with the above equation:

$$V_{in} = RCdV_C /dt + V_C$$

Recognizing that V_C is also V_{out}:

$$V_{in} = RCdV_{out} /dt + V_{out}$$

This equation is a simple first order differential equation with $\tau = RC$
so, replacing RC with τ and taking the Laplace transform,

$$V_{in} (s) = \tau sV_{out} (s) + V_{out}(s) \text{ ,}$$

$$\text{or } V_{out}(s) / V_{in} (s) = G(s) = 1/ (\tau s + 1)$$

Filter 2 above is two first order filters in series and can be modeled as the product of two first order transfer functions if negligable current flows from the first filter to the second. (we assumed that no current flows out of the original in the analysis). So, for filter 2, the transfer function is:

$$G_1(s) G_2(s) = [1/ (\tau_1 s + 1)] [1/ (\tau_2 s + 1)]$$

The magnitude of the frequency response is simply the magnitude of:

$$|G_1(s) G_2(s)|$$

and expressed in decibels, this would be:

$$20 \log (|G_1(s) G_2(s)|) \text{ or } 20 \log (|G_1(s)|) + 20 \log (|G_2(s)|)$$

This means we can simply add the magnitudes of the two filters when we are working with the magnitudes in decibels. For filters like those in filter 2, where the τ for both filters is the same ($R_1C_1 = R_2C_2$) the magnitude of the response is twice the magnitude of a single filter.

For filter 3, it may also be analyzed as two filters in series where the left hand side is a first order low pas filter. The filter on the right is a first order high pass filter. The transfer function of the high pass filter may be:

$$G(s) = \tau s / (\tau s + 1)$$

You may determine the magnitude response by substituting $i\omega$ for s and calculating the magnitude of G(s). The theoretical magnitude response of the filter would then be the sum of the responses of the first order low pass and the first order high pass.